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Prediction of rolling characteristics of ship models with the aid of a differential analyzer

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Cambridge 39, Mass.

Office of
G. C. Manning

Memo to: Capt. W. H. Buracker, USN.

From: Professor G. C. Manning

Date: 30 September 1946

Subj: Thesis work of LCDR T. E. Gerber, USN
LCDR R. Riley, USN.

The officers listed above have recently submitted a Master's Thesis done under my supervision entitled:

"The Prediction of the Rolling Characteristics of Ship Models with the Aid of a Differential Analyzer."

This is a study to determine the practicability of predicting a ship's behaviour at sea among waves from the statical stability curve and a rolling experiment on a model. The use of a mechanical integrator reduces the time to an amount to make such studies practical. Further study is required but the authors have pointed the way. An excellent thesis.

/s/ G. C. Manning

G. C. Manning
Professor of Naval Architecture

Cambridge, Massachusetts,
September 16, 1946.

Professor J. S. Newell,
Secretary of the Faculty,
Massachusetts Institute of Technology,
Cambridge, Massachusetts.

Dear Sir:

In accordance with the requirements for the degree of Master of Science in Naval Construction and Engineering, we submit herewith a thesis entitled, "Prediction of Rolling Characteristics of Ship Models with the Aid of a Differential Analyzer."

Respectfully,

[redacted]

Theodore E. Gerber,
Lieut. Commander, U. S. Navy.

[redacted]

Richard Riley,
Lieut. Commander, U. S. Navy.

PREDICTION OF ROLLING CHARACTERISTICS OF SHIP MODELS WITH THE AID OF A DIFFERENTIAL ANALYZER

By

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Lieut. Commander, U.S. Navy
B.S., U.S. Naval Academy, 1941

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Submitted in Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE IN NAVAL CONSTRUCTION

AND ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1946

Signature of Authors Theodore E. Gerber
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September 16, 1946

Signature of Thesis Supervisor

Signature of Chairman of
Department Committee on
Graduate Students

ACKNOWLEDGMENT

The authors wish to express their appreciation to Professor George C. Manning for his suggestion of the idea on which this thesis was based, and for his continued interest in its development. We also wish to express our indebtedness to Dr. Samuel H. Caldwell, of the Electrical Engineering Department of the Massachusetts Institute of Technology, whose constant assistance and advice in conducting this investigation was invaluable.

We also wish to thank Captain W. E. Kraft, U. S. Navy, Office of the Supervisor of Shipbuilding, New York City; Mr. Hollis Walters, of the Sperry Gyroscope Company; Mr. M. G. Forrest, of Gibbs and Cox; and Captain C. D. Wheelock, U. S. Navy, Professor of Naval Architecture, M. I. T., for their efforts to obtain data on actual ships rolling performance in a seaway.

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* From Serat (1)

SYMBOLS

A .... coefficient of $\frac{d\theta}{dt}$ in differential equation of rolling.

a .... height of wave, crest to hollow, inches.

B .... coefficient of $\left(\frac{d\theta}{dt}\right)^2$ in differential equation of rolling.

B .... beam of model, inches.

b .... block coefficient of model.

BM .... transverse metacentric radius, inches.

C .... a scale factor in the differential analyzer.

D .... a scale factor in the differential analyzer.

Δ .... a scale factor in the differential analyzer.

E .... a scale factor in the differential analyzer.

g .... acceleration of gravity, feet per second^2.

GM .... transverse metacentric height, inches.

GZ .... righting arm, inches.

K_1 .... coefficient of θ in Froude's equation for extinction of roll.

K_2 .... coefficient of θ^2 in Froude's equation for extinction of roll.

k .... radius of gyration of mass of model about a longitudinal axis through the center of gravity, inches.

L .... length of model, inches.

m .... midship section coefficient.

n_1, n_2, etc. .... gear ratios in the differential analyzer.

S .... wetted surface, square inches.

T .... complete rolling period of model, starboard to starboard, seconds.

T_1 .... complete period of wave, seconds.

t .... time, seconds.

x .... phase angle; angle by which inclination lags wave slope.

α .... coefficient of $\left(\frac{d\theta}{dt}\right)$ in rearranged differential equation used in integration.

β .... coefficient of $\left(\frac{d\theta}{dt}\right)^2$ in rearranged differential equation used in integration.

γ .... coefficient of GZ in rearranged differential equation used in integration.

Δ .... displacement of model, pounds.

θ .... transverse inclination of model, measured from vertical, radians.

λ .... wave length, inches.

μ .... height of metacenter above waterplane.

φ_m .... maximum wave slope.

φ .... slope of wave to horizontal.

ω .... circular frequency of sine wave.

SUMMARY

A. OBJECT.

The object of this investigation was to determine whether or not the differential analyzer affords a practical means for predicting the rolling characteristics of ship models.

B. METHOD.

From data on a series of models tested by M.E. Serat (1), coefficients for the differential equation of rolling were computed, scale factors for the differential analyzer were calculated, and actual integration of the rolling equation was carried out with the differential analyzer, using a basic machine diagram as shown in Figure I. Results from the analyzer were then compared with the actual data obtained by Serat in tabular form (see Results, Table I).

C. RESULTS.

Actual plots of angle of inclination from the vertical as a function of time were obtained directly from the analyzer (see "Results", Figure II and Table II).

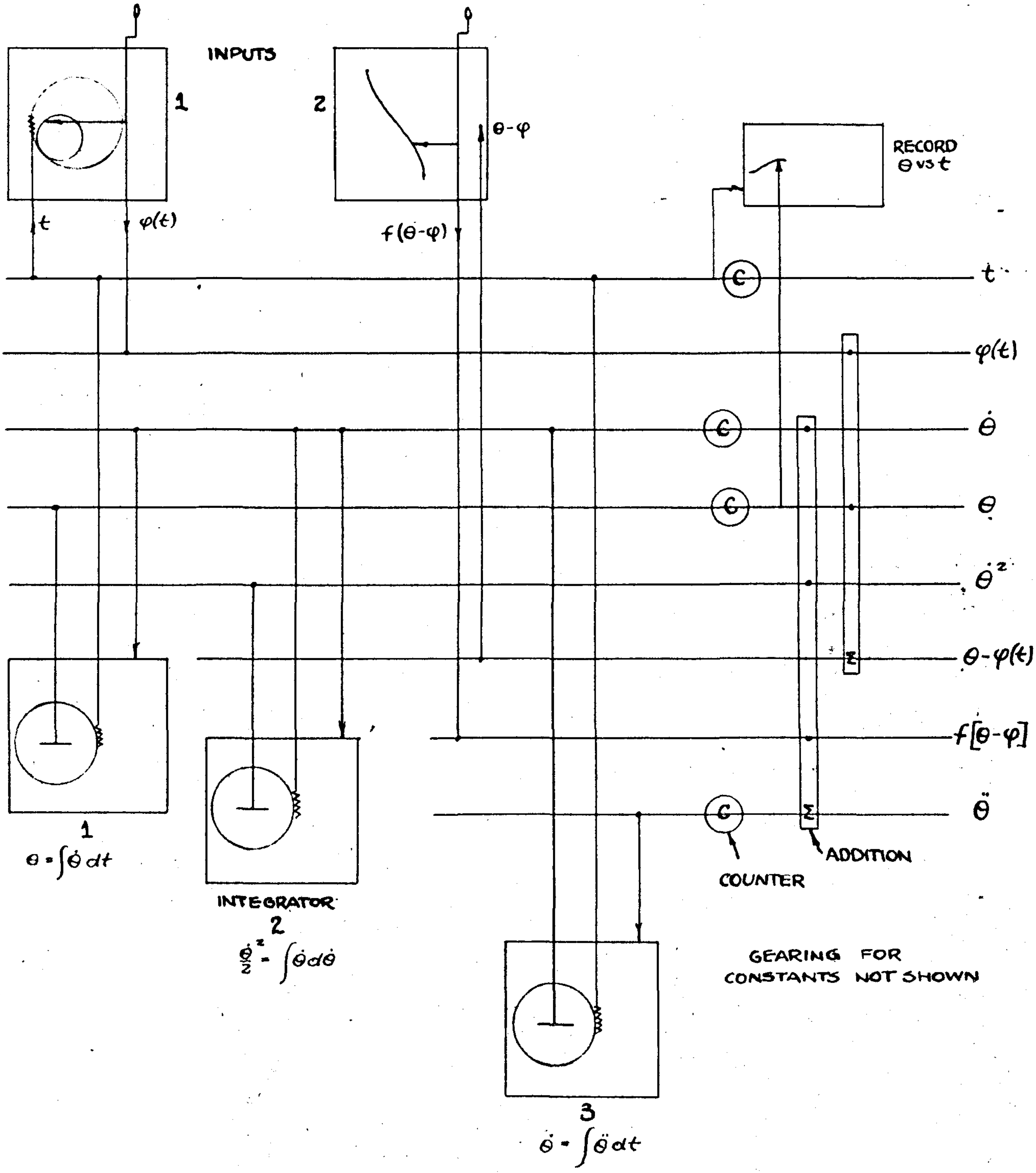
D. CONCLUSIONS AND RECOMMENDATIONS.

1. The method developed in this investigation of predicting the rolling characteristics of a model is fundamentally sound.

2. The method is also practical viewed from the standpoint of time required. A resume of time expended may be found in Results, Part B.

FIGURE I

BASIC
SCHEMATIC DIAGRAM - DIFFERENTIAL ANALYZER
FOR SOLUTION OF $\ddot{\theta} + \alpha\dot{\theta} + \beta\dot{\theta}^2 + \gamma f[\theta - \varphi(t)] = 0$



3. In addition, the method is also practical when viewed from the cost aspect. A resume of estimated charge for a single integration may be found in "Discussion of Results", Part C.

INTRODUCTION

The object of this investigation was to determine whether the differential analyzer affords a practical means for predicting the rolling characteristics of ship models. William Froude (2) proposed a method of solution of the differential equation of rolling by means of graphical integration. This method is quite lengthy and tedious, and as a result is rarely used. The differential analyzer installed at the Massachusetts Institute of Technology was designed for the solution of differential equations and is readily suited to the solution of the rolling equation. By the methods shown herein, a curve of angle of inclination versus time can be obtained with a limited amount of given data.

The investigation was undertaken on a model only, because it was found impossible to obtain full scale rolling data after several attempts in different sources.

PROCEDURE

The essential steps followed in this investigation were as follows:

1. Evaluation of coefficients in the differential equation of rolling with the aid of a paper prepared by M. E. Serat (1).

2. Evaluation of scale factors for the differential analyzer.

3. Setting up analyzer and actual integration.

4. Evaluation of results and comparison with test data collected by Serat.

For a detailed discussion of the procedure see Appendix A.

RESULTS

A. NUMERICAL RESULTS.

Table I below shows a comparison of Serat's observed data with that calculated under the following conditions:

Run 1. Normal-using complete rolling equation and wave slope at the surface.

Run 2. Reduced slope-using complete rolling equation and wave slope at a depth corresponding to the depth of the center of buoyancy in still water.

Run 3. Approximate-using rolling equation modified by neglecting the damping term in $\dot{\theta}^2$.

Run 4. Computed-from formulae given in Rossell and Chapman (3).

Also shown is the "phase angle" representing the angle by which the angle of inclination lags the wave slope.

Table II is a representative section of the recording tape of the differential analyzer.

Table III contains the values of θ versus time.

Figure II is a tracing of the plot made by the differential analyzer during the same cycle in which the values in Table III were recorded.

TABLE I

COMPARISON OF MAXIMUM VALUES

Item	Units	Observed	Run 1	Run 2	Run 3	Computed
		CONDITION				
θ_{max}	deg.	14.7	15.20	14.19	15.25	16.2
$\dot{\theta}_{max}$	rad/sec	----	1.38	1.30	1.39	1.47
$\ddot{\theta}_{max}$	rad/sec^2	----	7.42	6.80	7.52	7.74
Phase angle	deg.	----	63-10	65-51	63-10	78-20
ϕ_{max}	deg.	----	2.67	2.47	2.67	2.67
Period	sec.	1.20	1.200	1.199	1.204	1.200

TABLE II

TYPICAL SECTION OF RECORDING TAPE

1000t	2.66 $\ddot{\theta}$	100	14.28 $\dot{\theta}$
08798	00016	99853	00195
08996	99997	00009	00197
09198	99978	00168	00196
09398	99959	00323	00193
09596	99940	00473	00187
09796	99923	00620	00180
09996	99906	00760	00171
10198	99890	00893	00160
10396	99875	01015	00147
10596	99860	01126	00133
10798	99847	01227	00117
10998	99835	01314	00100
11196	99824	01386	00082
11396	99815	01443	00062
11598	99808	01485	00042
11796	99803	01510	00021
11996	99802	01518	00000
12198	99802	01510	99979

These readings were taken during Run 1.

Note: Counters read to nearest tenth of turn.

TABLE III

VALUES OF θ VS t FOR FIGURE II

Run 1		Run 2		Run 3	
t	θ	t	θ	t	θ
0.000	00.00	0.000	00.00	0.000	00.00
0.001	0.09	0.018	1.29	0.015	1.17
0.041	3.23	0.058	4.17	0.035	2.75
0.081	6.20	0.098	6.87	0.075	5.78
0.121	8.93	0.138	9.27	0.115	8.57
0.161	11.26	0.178	11.30	0.155	11.60
0.201	13.14	0.218	12.84	0.195	12.95
0.241	14.43	0.258	13.83	0.235	14.34
0.281	15.10	0.298	14.20	0.275	15.10
0.321	15.10	0.338	13.94	0.295	15.23
0.361	14.42	0.378	13.06	0.302	15.24
0.401	13.09	0.418	11.60	0.322	15.19
0.441	11.18	0.458	9.63	0.362	14.57
0.481	8.79	0.498	7.25	0.402	13.30
0.521	6.02	0.538	4.58	0.442	11.44
0.561	3.01	0.578	1.71	0.482	9.07
0.601	-0.12	0.598	0.25	0.522	6.33
0.636	-2.86	0.638	-2.69	0.563	3.33
0.671	-5.50	0.678	-5.50	0.603	0.19
0.711	-8.31	0.718	-8.07	0.623	-1.39
0.731	-9.59	0.758	-10.05	0.663	-4.48
0.771	-11.82	0.778	-11.04	0.702	-7.37
0.811	-13.56	0.807	-12.42	0.742	-9.96
0.851	-14.71	0.827	-13.10	0.783	-12.14
0.891	-15.21	0.867	-13.95	0.823	-13.79

TABLE III (cont.)

t	Q	t	Q	t	Q
0.931	-15.04	0.907	-14.18	0.863	-14.85
0.971	-14.19	0.942	-13.85	0.883	-15.13
1.011	-12.71	0.977	-13.05	0.903	-15.25
1.051	-10.67	1.017	-11.58	0.923	-15.19
1.091	-8.15	1.058	-9.58	0.962	-14.57
1.131	-5.32	1.098	-7.20	1.003	-13.28
1.171	-2.25	1.137	-4.52	1.042	-11.43
1.191	-0.69	1.177	-1.65	1.083	-9.06
1.200	0.00	1.199	0.00	1.122	-6.33
				1.162	-3.32
				1.202	-0.17
				1.204	0.00

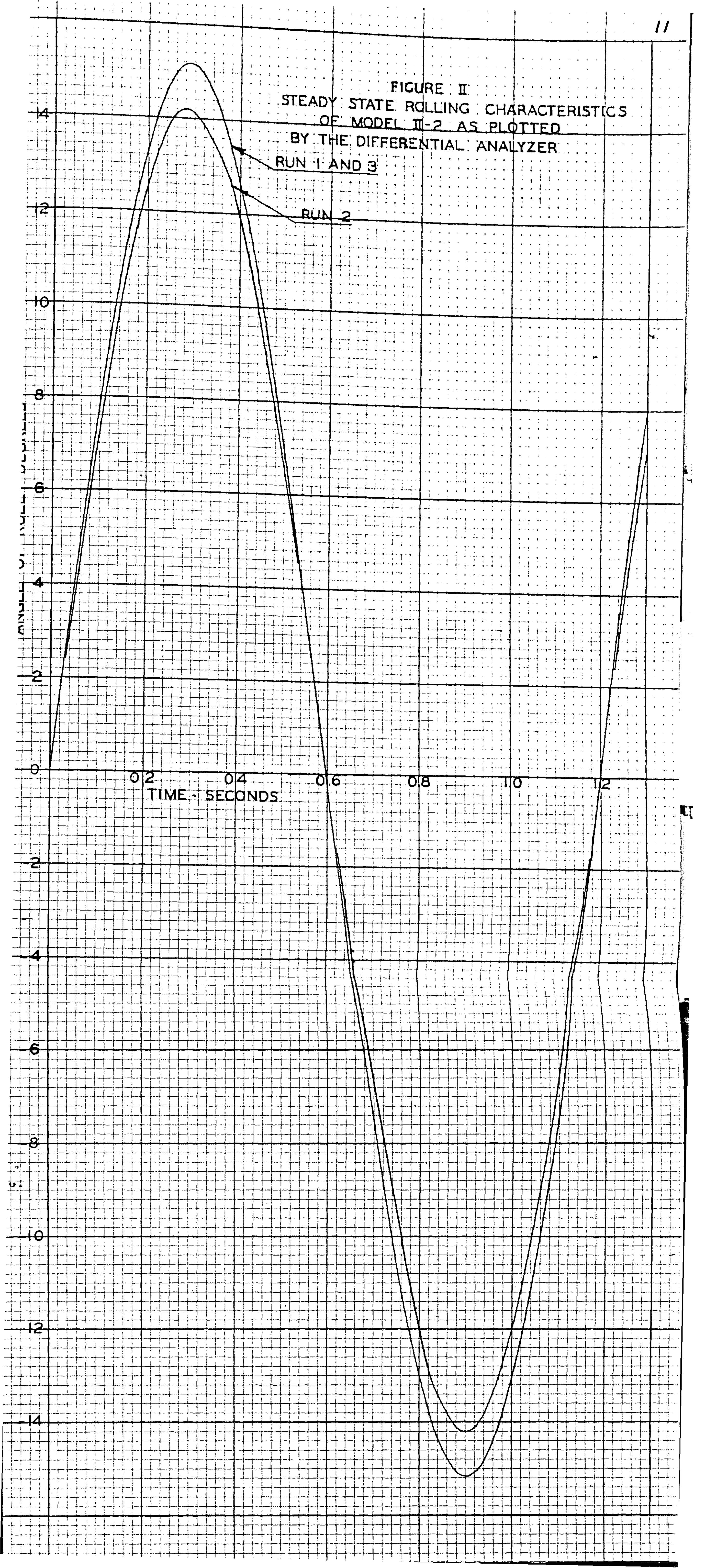


FIGURE II
STEADY STATE ROLLING CHARACTERISTICS
OF MODEL II-2 AS PLOTTED
BY THE DIFFERENTIAL ANALYZER
RUN 1 AND 3
RUN 2
TIME - SECONDS
0.2
0.4
0.6
0.8
1.0
1.2
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
-12
-14

B. TIME EXPENDED.

An estimate of time expended in obtaining the above results follows:

1. Evaluation of coefficients in the differential equation of rolling	3 hours
2. Evaluation of scale factors and gear ratios in the differential analyzer	6 hours
3. Plotting curves for input tables	3 hours
4. Setting up analyzer	16 hours
5. Actual integration	30 hours
6. Evaluation of results	4 hours
Total time expended	62 hours

C. APPROXIMATE COST.

The cost involved in attaining the results was approximately as follows:

1. Reservation charges on differential analyzer, 3 days at $20.00 per day	$60.00
2. Hourly charges on integrators, hours on 3 units at $0.25 per unit per hour	22.50
3. Labor charges, 24 hours for two operators at $1.60 per hour	76.80
	$159.30

DISCUSSION OF RESULTS

A. NUMERICAL.

1. The accuracy of results is such that they can be assumed reliable to tenths of degrees so that there is a difference of a half degree between experimental data and Run 1. Serat estimated that the roll would be 16.0 degrees by a method given in his article. Thus the analyzer gave a closer estimate. Errors remaining between 15.2 and 14.7 degrees, or 3.4% error, are probably due to errors in damping data itself, inability to read damping curve to that accuracy, or a combination of both. That part due to errors in the analyzer was limited to 0.4%.

2. The use of reduced wave slope yields the same error but in the opposite direction so that no conclusive statement can be made. However, the method used for determining the restoring force, (i.e. $(\theta - \phi)$, or wedge angle) does not justify using a slope less than that at the surface.

3. The use of a modified differential equation also yields nothing on which to base a definite conclusion other than with damping terms of relative values as used it is unnecessary to include the term in $\dot{\theta}^2$. For other values this term probably can become important. It is of interest to note that the phase angle was the same in Runs #1 and #3.

4. The phase angle relationship did not check at all well, and this is readily explained by the fact that the

formula given in Rossell and Chapman (3) where x is the phase angle :

$$x = \tan^{-1} \frac{2K_1}{\pi} \cdot \frac{\frac{T}{T_1}}{1-\left(\frac{T}{T_1}\right)^2} \qquad (1)$$

can be transformed to

$$x = \tan^{-1} \frac{2\pi A}{T_1} \cdot \frac{1}{\Delta GM - \frac{\Delta k^2}{g}\frac{4\pi^2}{T_1^2}} \qquad (2)$$

This formula was derived from the assumption that righting arm is a linear function of Θ. In the case analyzed the righting arm at points above 8 degrees was considerably above the line whose tangent is GM, as much as 6.7% above at 16 degrees. Since the terms in the denominator of (2) are of the same order of magnitude, a change in GM of less than 6% can change the tangent of the phase angle enough to give an x of the value attained above, or about 63 degrees.

B. TIME.

The record of time expended, given in "Results B", does not, of course, reflect the true value of the method. The authors at the outset were unfamiliar with the analyzer and the methods used in its operation. A person experienced in the use of the analyzer would obviously require much less time. Below is an estimate of the time that should be required for a methodical attack on this problem, providing thorough data has been made available:

1.	Evaluation of coefficients	1 hour
2.	Evaluation of scale factors and gear ratios	3 hours
3.	Plotting curves for inputs	3 hours
4.	Setting up analyzer	8 hours
5.	Actual integration	½ hour per wave
6.	Evaluation of results	1 hour
	Total time required	16½ hours

If this type of problem were always worked on the analyzer the set-up time would be reduced from 8 hours to about 3 hours, and the scale factor computations and plotting could be done in about 2 hours apiece. Thus, the total would be reduced to about 9½ hours.

C. COST.

As with time, the cost is in excess of that which would hold for an investigator more experienced with the problem and this type of solution. An estimate of cost for an integration for one ship or model, and for three or four sea conditions, is as follows:

1.	Reservation charge	$20.00
2.	Labor	25.60
3.	Integrator charges	6.00
	Total cost	$51.60

CONCLUSIONS

A.

The method herein described does afford a practical means of solving the differential equation of rolling both from the standpoint of time and cost, assuming that the results obtained are considered of importance to the designer. It would seem that if characteristics of this nature were desired at all, this would be the logical approach. It is obviously more direct, accurate, and less tedious than the graphical integration proposed by Froude (2).

B.

It is also concluded that for cases where K_2 is less than 4% of K_1, the solution can be obtained within 0.3% accuracy with the term in $\dot{\theta}^2$ omitted. It appears to have no effect on the magnitude of roll, velocity, acceleration, or even phase angle, when restricted to the value above.

RECOMMENDATIONS

A.

It is felt that this method deserves further investigation with the possibility of practical use in connection with preliminary design. It does not seem impractical that this type of solution could be accomplished on a special type of differential analyzer built for the specific purpose of solving only the rolling equation so that a naval architect could have his own apparatus. This, of course, would depend on the importance that the naval architect attaches to this more elaborate method of investigating a design's sea-keeping ability. Perhaps some sort of standard waves could be established to give a basis of comparison among several ships, much as is now done in longitudinal strength.

B.

The extension of this method is hampered by lack of rolling data, and not a great deal of information of ship's damping is available. These have probably been neglected heretofore due to the fact that there has been no particular use for such data. However, if this method were put into practice, such data could be obtained and investigations made to correlate it to a more advanced point than is now the case.

APPENDIX

APPENDIX A

DETAILS OF PROCEDURE

1. Evaluation of coefficients in the differential equation of rolling.

The differential equation of rolling used for integration was:

$$\frac{k^2\Delta}{g}\ddot{\theta}+A\dot{\theta}+B\dot{\theta}^2+\Delta GZ=0 \qquad (1)$$

Before integration it was necessary to evaluate g, k, Δ, A, B, and GZ. The value of g was taken at 386 inches per (second)^2. The paper of Serat (1) gives information on several models of the parallel middle bodies of ships. One of these models, II-2 with bilge keels installed, was selected for study. Its characteristics are given in Figures III, IV, and V, and Table IV. The value of was given but other terms were found by less direct methods. In order to get A and B it was necessary to convert the curves given by Serat of the rate of angular damping versus angle of roll for rolling in still water to a curve of extinction. This was done by multiplying the rate of angular damping at a given angle by the period of roll in still water and plotting this decrement against the angle of roll. Having the curve of extinction, its equation was approximated by:

$$\delta\theta = K_1\theta + K_2\theta^2 \qquad (2)$$

and values of K_1 and K_2 were evaluated. It so happened that the curve could have been approximated by the formula:

$$\delta\theta = K_3\theta^n$$

FIGURE III

GENERAL FORM OF MODEL II-2

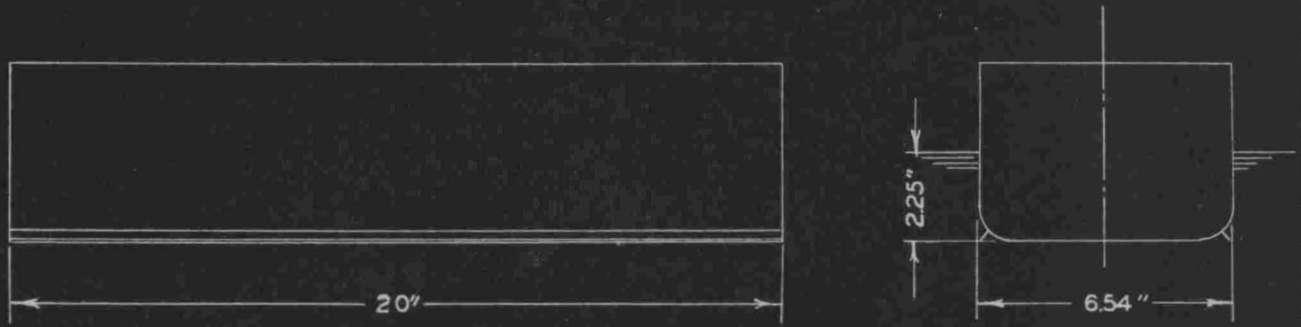


TABLE IV
MODEL DATA

DISPLACEMENT	10.21 LB.
LENGTH	20.00 IN.
BEAM	6.54 IN.
DRAFT	2.25 IN.
MIDSHIP-SECTION AREA	14.14 SQ.IN.
MIDSHIP-SECTION COEFFICIENT	0.96
WIDTH OF BILGE KEELS	3/16"
WETTED SURFACE	201.6 SQ.IN.
HEIGHT OF METACENTER ABOVE W.L.	0.67 IN.
METACENTRIC RADIUS	1.76 IN.
BEAM-DRAFT RATIO	2.91

FIGURE IV
DAMPING CURVES MODEL II-2
WITH BILGE KEELS
GM = 0.496" T = 1.18 SEC.

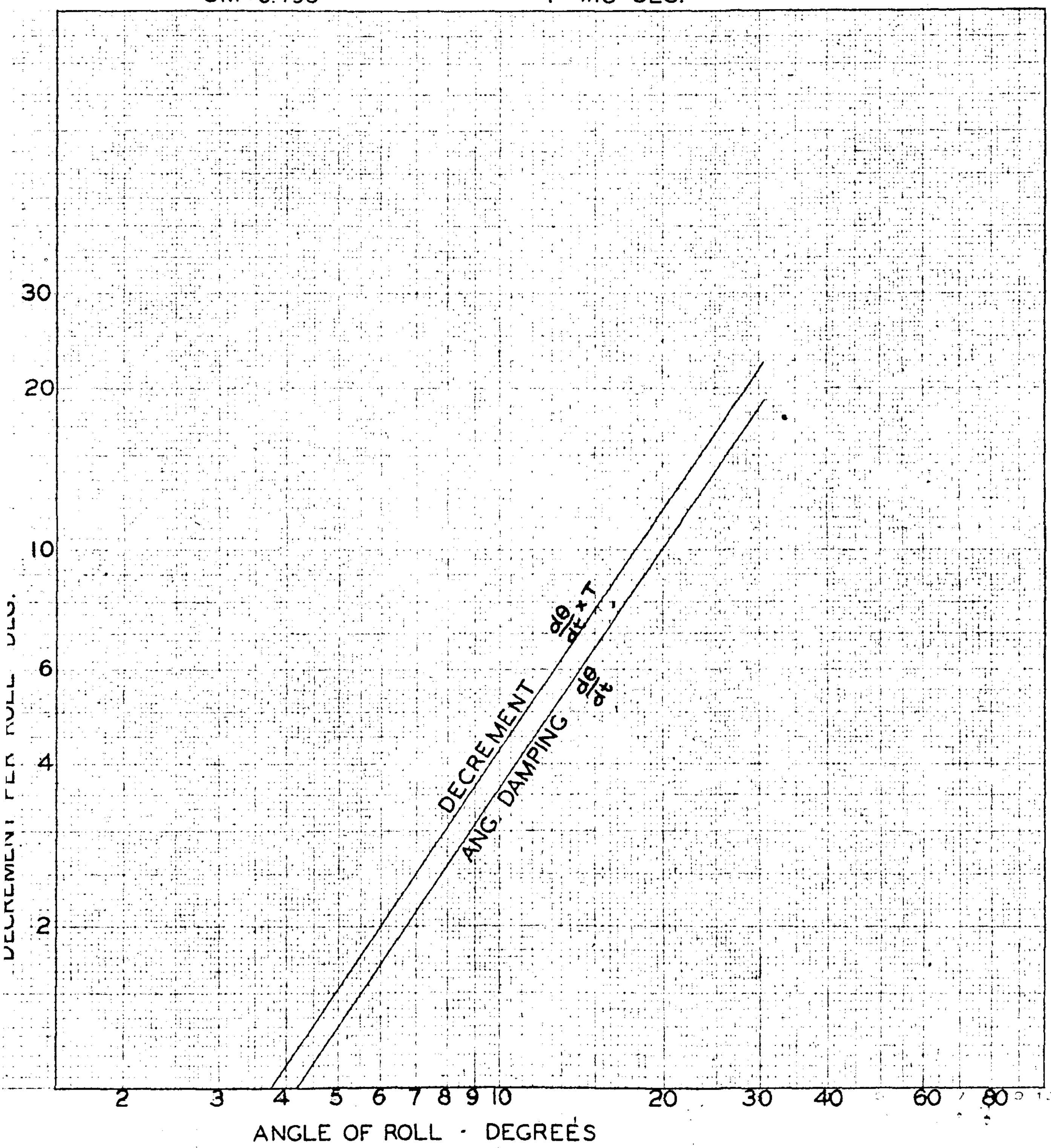
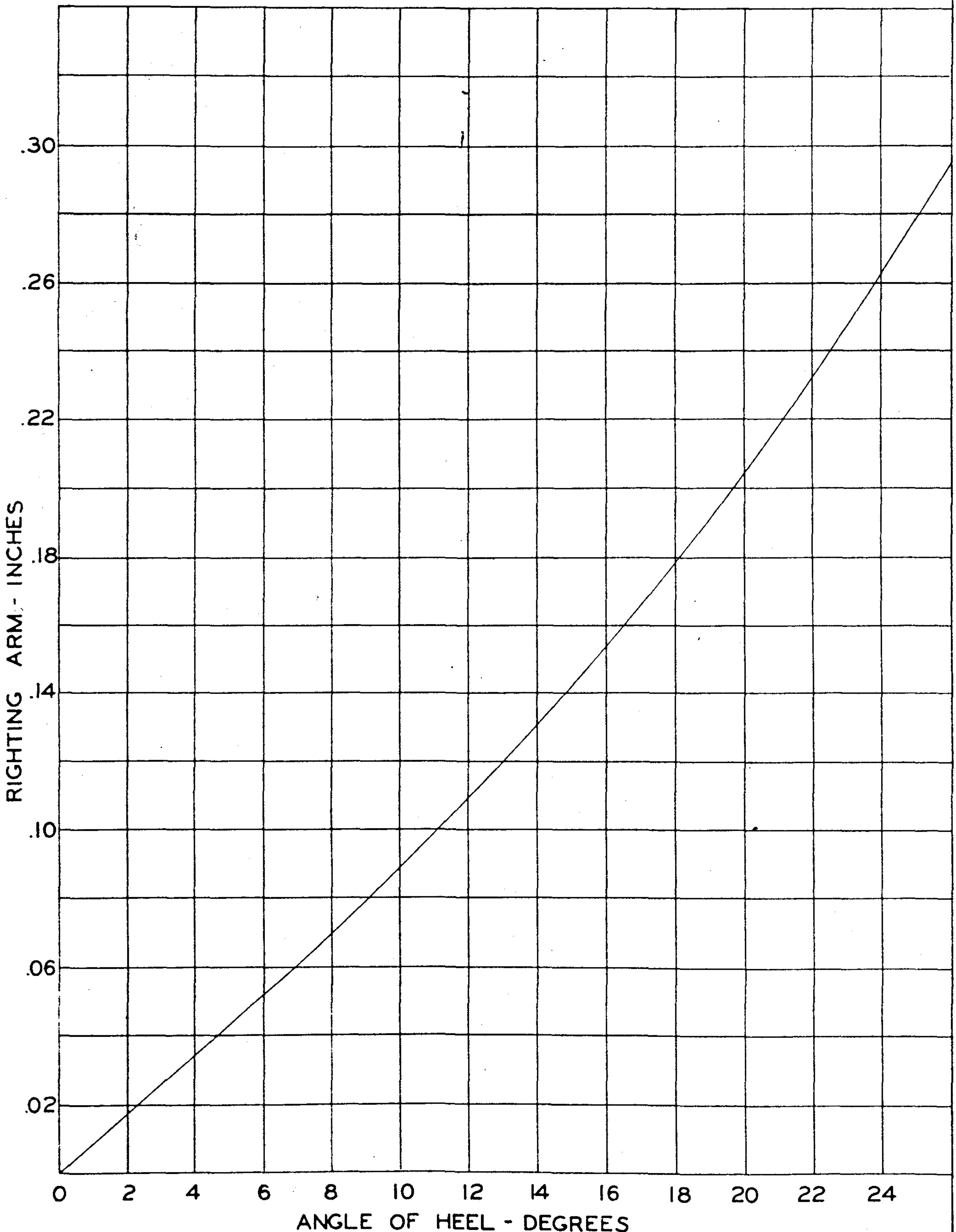


FIGURE V
CURVE OF RIGHTING ARM
MODEL II-2
GM .496 IN.



However, ease in use of the differential analyzer made it more feasible to use the first equation given. With the aid of the following equations from Rossell and Chapman (3), the values of A and B were obtained once K_1 and K_2 were found.

$$K_1 = \frac{\pi^2 A}{T_r \Delta GM} \qquad (3)$$

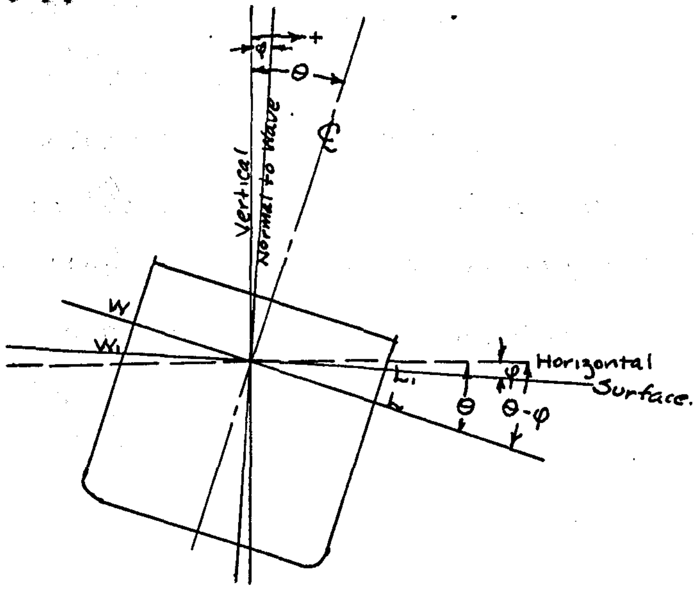
$$K_2 = \frac{16\pi^2 B}{3T_r^2 \Delta GM}. \qquad (4)$$

In these equations the period of resisted rolling in still water, T_r, and metacentric height, were given by Serat from his still water rolling test.

Also from K_1, T_r, and with formulae given in Rossell and Chapman (3), the value of k was found

$$T_r = \frac{1.108\,k}{\sqrt{GM}} \cdot \frac{1}{\sqrt{1-\frac{K_1^2}{\pi^2}}} \qquad (5)$$

Finally, the righting arm, GZ, is not a constant in the integration and was considered to be a function of the immersed wedge angle which is the difference between the angle of inclination from the vertical, θ, and the normal to the wave slope, ϕ, as shown below:



The statical stability curve, GZ versus θ, in still water which is actually righting arm versus wedge angle, was used as an input to the analyzer with the argument $\theta - \phi$, and referred to hereafter as $f(\theta - \phi)$.

Wave slope, ϕ, was considered a sine function of time and the maximum value was computed from:

$$\varphi_m = \frac{\pi a}{\lambda} \quad \text{radians} \qquad (6)$$

A curve of ϕ versus t was used as an input to the analyzer.

2. <u>Evaluation of scale factors and gear ratios.</u>

Since the differential analyzer is a mechanical apparatus it has definite physical limits. In order not to overrun these limits, and in order to use them to best advantage, estimated extreme values of all variables involved must be selected and scale factors assigned to all variables.

A diagram such as Figure VI following was drawn up and extreme values estimated and called

$$\phi_m, \ \ddot{\theta}_m, \ \dot{\theta}_m, \ \theta_m, \ (\theta - \phi)_m, \text{ and } f(\theta - \phi)_m$$

The differential equation (1) was transposed so that

$$\ddot{\theta} + \alpha\dot{\theta} + \beta\dot{\theta}^2 + \gamma f(\theta - \varphi) = 0 \qquad (7)$$

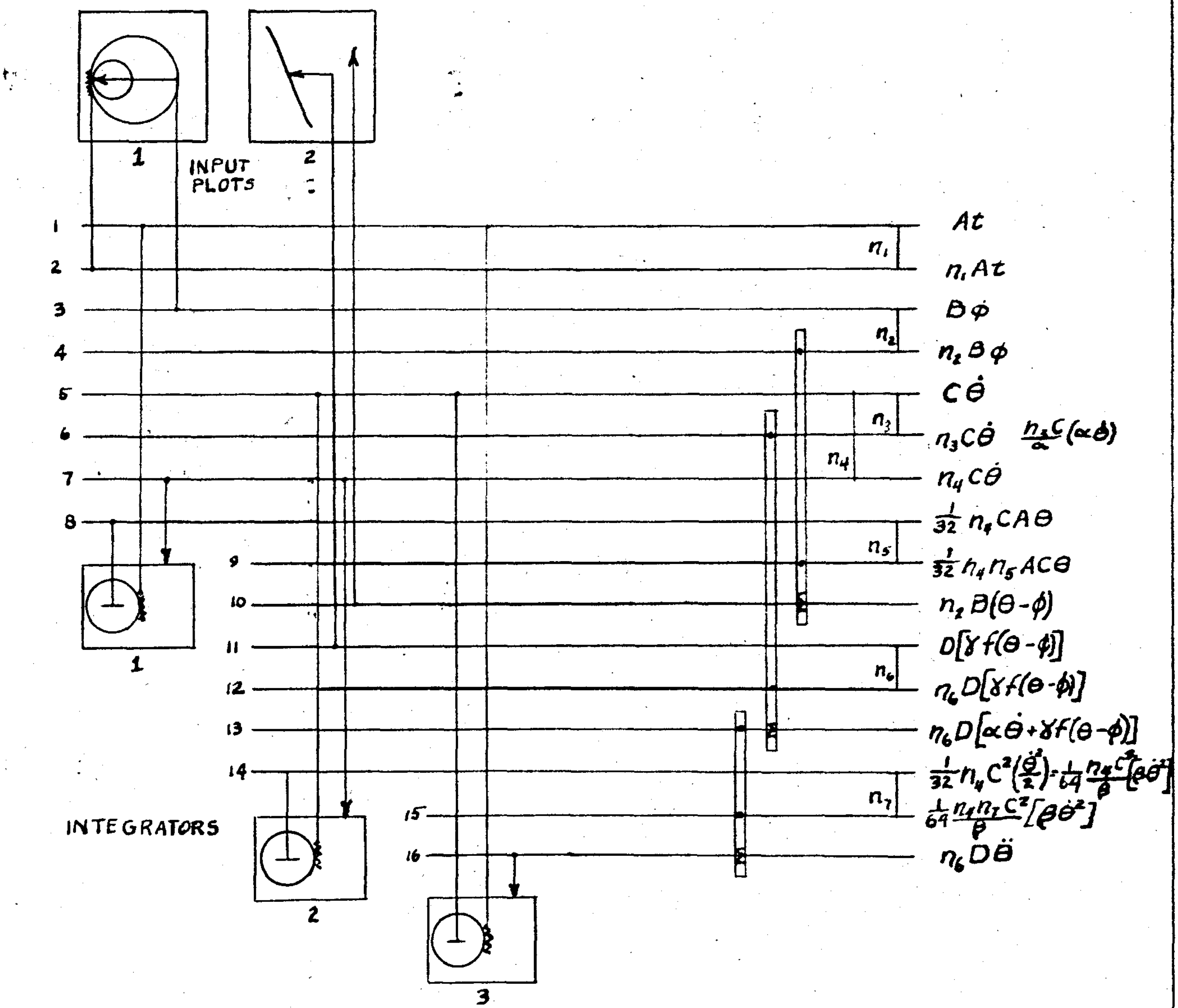
where

$$\alpha = \frac{Ag}{\Delta k^2}; \quad \beta = \frac{Bg}{\Delta k^2}; \quad \gamma = \frac{g}{k^2}$$

Gearing and scale factors were denoted by letters as shown, and their values were computed from certain expressions that can be written. Throughout the calculations,

FIGURE VI
SCHEMATIC DIAGRAM USED IN SCALE FACTOR AND GEAR RATIO COMPUTATIONS

SCALE FACTORS A, B, C, D
GEAR RATIOS $n_1, n_2, \cdots n_7$



scale factors refer to the number of turns of a given shaft to represent one unit of the variable. For example, the time shaft makes A turns to represent one unit of time. Gear ratios are given as the ratio of driven shaft turns to driven turns. Integrators produce the product of the scale factors of the inputs divided by 32. For example, Integrator #1 takes $n_4C\dot{\theta}$ and At, and produces $\frac{n_4C\theta A}{32}$. With these facts known, all variables with appropriate scale factors were written at the end of each shaft as shown. The equalities and inequalities were set up as follows:

(a) Since the polar plot of ϕ versus t makes one turn for one hundred turns of the input shaft and the period of the wave was known

$$n_1 At \times T_1 = 100 \tag{8}$$

(b) Since the polar plot output follower has a limit of $\pm$ 180 turns from midposition

$$B \times \varphi_m \leqslant 180 \tag{9}$$

(c) Input plot 2 was on a table with a range of the independent variable of 480 turns, and of the dependent variable of 360 turns. Hence

$$n_2 B \times 2(\theta-\varphi)_m \leqslant 480 \tag{10}$$

$$D \times 2\left[\gamma f(\theta-\varphi)_m\right] \leqslant 360 \tag{11}$$

(The factor 2 resulting from the fact that both variables may be plus or minus.)

(d) The integrators have an excursion of 40 turns to either side of their midposition, hence

$$n_4 C \dot{\theta}_m < 40 \tag{12}$$

$$n_6 D \ddot{\theta}_m < 40 \tag{13}$$

(e) The equalities from the adder gears are:

$$n_2 B = \frac{1}{32} n_4 n_5 AC \tag{14}$$

$$n_6 D = \frac{n_3 C}{\alpha} = \frac{1}{64} n_4 n_7 \frac{C^2}{\beta} \tag{15}$$

$$C = \frac{1}{32} n_6 AD. \tag{16}$$

Solving these equations by trial and error, the gear ratios as shown in Figures VII were arrived at.

Gearing for record of θ versus to and their counters was arranged to give values easily read, as 100, 1000t, etc.

3. <u>Setting up analyzer and actual integration.</u>

The analyzer was arranged as shown in Figure VII, and the first integration, hereafter referred to as Run 1, was carried out with the wave slope of the surface. The maximum angle of roll in the steady state and the phase relationship between wave slope, or exciting force and resulting oscillation was estimated from:

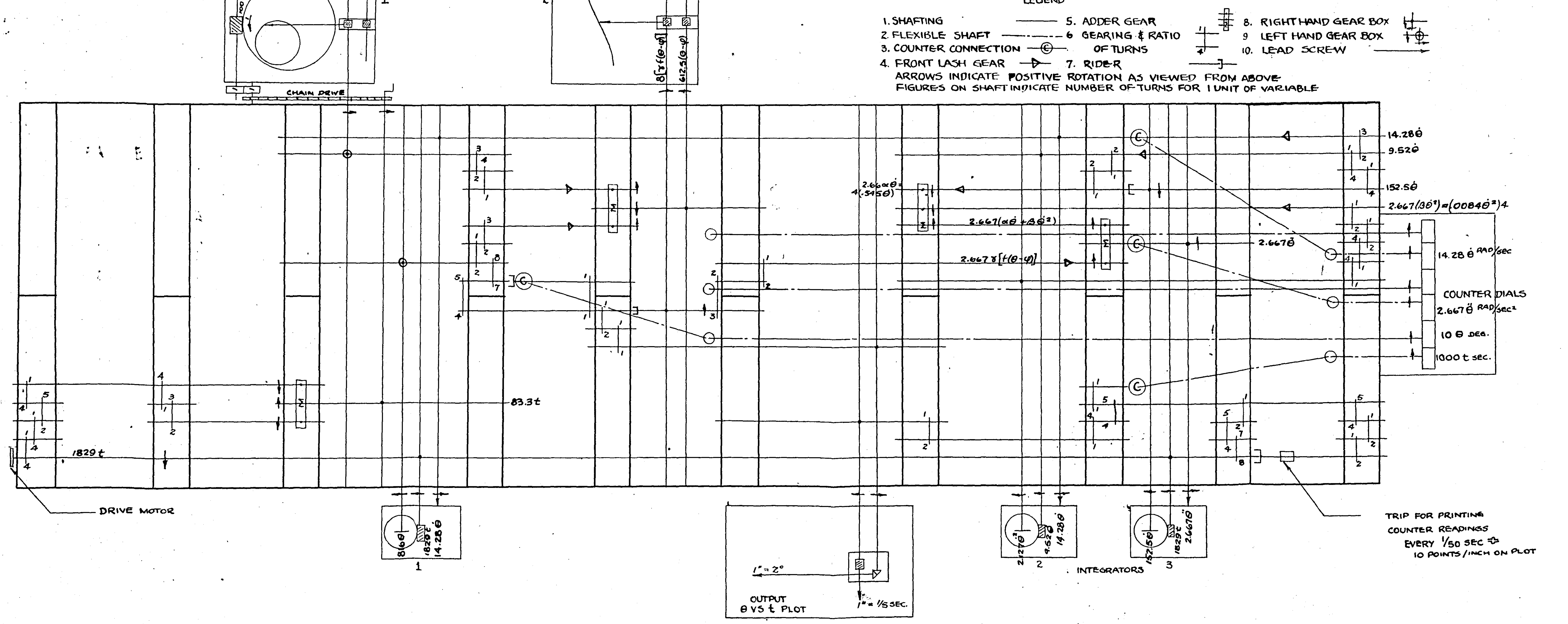
$$\theta = \frac{\varphi_m \sin\left(\frac{2\pi t}{T_1} - x\right)}{\left[\left(1 - \frac{T^2}{T_1^2}\right)^2 + \frac{4K_1^2}{\pi^2}\left(\frac{T}{T_1}\right)^2\right]^{1/2}} \tag{17}$$

where

$$x = \tan^{-1} \frac{2K_1}{\pi} \frac{\frac{T}{T_1}}{1 - \left(\frac{T}{T_1}\right)^2} \tag{18}$$

which are given in Rossell and Chapman (3).

FIGURE VII
SCHEMATIC DIAGRAM
DIFFERENTIAL ANALYZER SET-UP



With these values initial conditions were set up where t was zero, angle of inclination was θ computed from (17), the velocity of the roll was zero, the wave slope was leading the angle of roll by x degrees and the acceleration was the negative of $f(\theta - \phi)$. The problem was run until differences between successive inclinations were less than 0.4% for several complete cycles. This was assumed to be the steady state. Tapes recording counter readings approximately 50 times per cycle and record of θ versus t plotted by the analyzer were made during the run and were marked when the steady state was reached.

Following the same general procedure Run 2 was made using a wave slope corresponding to motion of wave particles at the height of the center of buoyancy. This was done by considering that the wave slope decreases as an exponential from the surface so that

$$\varphi' = e^{-\frac{2\pi d}{\lambda}} \varphi_m \sin \frac{2\pi t}{T_1} \qquad (19)$$

where d is the depth below the surface to the center of buoyancy and ϕ_m is the maximum value of the wave slope at the surface given in equation (6).

The final integration, Run 3, was made with the surface wave slope plot, but with the #2 Integrator inoperative, so that

$$\ddot{\theta} + \alpha\dot{\theta} + \gamma f(\theta - \varphi) = 0 \qquad (20)$$

was being integrated. This was done to show the effect of the $\dot{\theta}^2$ term, which is often neglected.

In all runs the phase angle at steady state was recorded at the end of the run.

4. Evaluation of results.

The plots of Θ versus t were used only as a guide to the general trend of the integration and results were taken from printed record of counter readings. Counter readings or turns made were converted to the actual values of the variables by application of the appropriate scale factor. Maximum values of inclination were compared with the estimates and experimental values given by Serat (1).

A record was prepared of time spent on various phases of the work so as to contrast this with the other methods of estimating or computing roll to be expected from a ship form among waves.

APPENDIX B

SAMPLE CALCULATIONS

1. Evaluation of coefficients.

The curve of angular damping in Figure IV was converted to a curve of extinction by multiplying by 1.18 seconds.

When $\theta = 12^\circ \qquad \delta\theta = 5.52\,^\circ/\text{roll}$

$\theta = 16^\circ \qquad \delta\theta = 8.42\,^\circ/\text{roll}$

By equation (2)

$$12K_1 + 144K_2 = 5.52$$
$$16K_1 + 256K_2 = 8.42$$

Solving simultaneously

$$K_1 = 0.262 \qquad K_2 = 0.0165$$

From equations (3) and (4)

$$A = \frac{1.18(.262)(10.21)(496)}{\pi^2} = 0.158 \text{ lb-in-sec}$$

$$B = \frac{(1.18)^2\,3(.0165)(10.21)(.496)}{16\pi^2} = 0.0022 \text{ lb-in-sec}^2$$

By equation (5)

$$k = \frac{1.18\sqrt{.496}}{1.108}\sqrt{1 - \frac{(.262)^2}{\pi^2}} = 2.59 \text{ in}$$

$$\therefore \quad \alpha = \frac{Ag}{\Delta k^2} = \frac{(.158)(386)}{(10.21)(2.59)^2} = 0.893\ {}^1/\text{sec}$$

$$\beta = \frac{Bg}{\Delta k^2} = \frac{(.0022)(386)}{(10.21)(2.59)^2} = 0.0124$$

$$\gamma = \frac{g}{k^2} = \frac{386}{(2.59)^2} = 57.5\ {}^1/\text{in sec}^2$$

By equation (6) and using wave of 1.39 inches height and 94 inches length

$$\varphi_m = \frac{\pi(1.39)}{94} = 0.0464 \quad \text{radians or } 2.67^\circ$$

For the wave of reduced slope:

From Table IV $d = 1.16$ inches

From equation (19)

$$\varphi'_m = e^{\frac{-2\pi(1.16)}{94}}(.0464) = 0.0430 \text{ radians or } 2.47^\circ$$

2. Evaluation of scale factors and gear ratios.

Maximum value of Θ expected:

$$\Theta_m = \pm 20^\circ \text{ or } \pm 0.35 \text{ rad.}$$

If $\Theta \approx \Theta_m \sin\omega t$ with period of 1.20 seconds

then

$$|\dot{\Theta}_m| = \omega\Theta_m$$

$$|\ddot{\Theta}_m| = \omega^2\Theta_m$$

$$\text{and } \omega = \frac{2\pi}{T} = \frac{2\pi}{1.20} = 5.24$$

Therefore

$$|\dot{\Theta}_m| \approx 1.83 \text{ rad/sec}$$

$$|\ddot{\Theta}_m| \approx 9.55 \text{ rad/sec}^2$$

use

$$\Theta_m = \pm 0.35 \text{ rad.} \qquad \varphi_m = \pm .06 \text{ rad}$$

$$\dot{\Theta}_m = \pm 2.5 \text{ rad/sec}$$

$$\ddot{\Theta}_m = \pm 13.0 \text{ rad/sec}^2$$

At $(\Theta - \phi)_m = \pm .41$, from stability curve.

$$f(\Theta - \varphi)_m = \pm 0.38 \text{ in}$$

Substituting these in equations (8) through (16) and by trial and error the following were established:

$A = 1829$

$B = 2450$

$C = 152.4$

$D = 8.0$

$n_1 = 0.04555 = \frac{5}{128}(1 + \frac{1}{6})$

$n_2 = 1/6$

$n_3 = 1/64$

$n_4 = 3/32$

$n_5 = 1/2$

$n_6 = 1/3$

$n_7 = 1/1024$

3. <u>Setting up analyzer and actual integration.</u>

Since the wave slope was considered to be a sinusoidal function of time the curve on the polar plot was a circle. This plot rotated once for each 1.2(1829) turns of the t shaft, thus representing a sine function with a period of 1.20 seconds since 1829 turns were equivalent to one second. The output was 2450 ϕ_m. To represent a maximum wave slope of 0.0464 radians would take 2450(0.0464) turns. With a follower making 20 turns per inch the circle should be made to have a radius of

$$\frac{2450(0.0464)}{40} = 4.27 \text{ inches}$$

The input on table #2 was plotted by the follower to be used in the runs in order to avoid errors inherent in the lead screws. Points were selected from Figure V, converted to turns by 612.6(θ - ϕ) and 8 γ f(θ - ϕ).

Estimated maximum angle of roll and phase angle were from equations (17) and (18).

$$\Theta = \frac{0.0464}{\sqrt{\left(1-\frac{1.18^2}{1.20^2}\right)^2 + \frac{4(.262)^2}{\pi^2}\left(\frac{1.18}{1.20}\right)^2}} = 0.283 \text{ rad}$$

or $16.2°$

$$x = \tan^{-1} \frac{2(.262)}{\pi} \frac{\frac{1.18}{1.20}}{1-\left(\frac{1.18}{1.20}\right)^2} = \tan^{-1} 4.8$$

$$x = 78°20'$$

APPENDIX C

ORIGINAL DATA

Data as actually recorded by the analyzer is filed with the Center of Analysis, Massachusetts Institute of Technology.

APPENDIX D

BIBLIOGRAPHY

(1) Serat, M. E., "Effect of Form on Roll", Society of Naval Architects and Marine Engineers Transactions, 1933, Volume 41, pages 160-180.

(2) Froude, William, "The Graphic Integration of the Equation of Ship's Rolling", Institution of Naval Architects, 1875.

(3) Rossell and Chapman, "Principles of Naval Architecture", Volume II, Chapter 1, New York, 1942.